The current issue and full text archive of this journal is available at
http://www.emeraldinsight.com/0961-5539.htm

Received September 2001
Accepted March 2002

Effect of outlet positions and various turbulence models on mixing in a single and multi strand tundish

Pradeep K. Jha and Sukanta K. Dash
Department of Mechanical Engineering, Indian Institute of Technology, Kharagpur, India

Keywords *Turbulence, Model, Containers, Flow*

Abstract *The Navier-Stokes equation and the species continuity equation have been solved numerically in a boundary fitted coordinate system comprising the geometry of a large scale industrial size tundish. The solution of the species continuity equation predicts the time evolution of the concentration of a tracer at the outlet of a single strand bare tundish. The numerical prediction of the tracer concentration has been made with three different turbulence models; (a standard* k-ε, *a* k-ε *RNG and a Low Re number Lam-Bremhorst model) which favorably compares with that of the experimental observation for a single strand bare tundish. It has been found that the overall comparison of* k-ε *model with that of the experiment is better than the other two turbulence models as far as gross quantities like mean residence time and ratio of mixed to dead volume are concerned. However, it has been found that the initial transient development of the tracer concentration is best predicted by the Lam-Bremhorst model and then by the RNG model. The* k-ε *model predicts the tracer concentration much better than the other two models after the initial transience (*$t > 40$percent *of mean residence time) and the RNG model lies in between the* k-ε *and the Lam-Bremhorst one. The numerical study has been extended to a multi strand tundish (having 6 outlets) where the effect of outlet positions on the ratio of mix to dead volume has been studied with the help of the above three turbulence models. It has been found that all the three turbulence models show a peak value for the ratio of mix to dead volume (a mixing parameter) when the outlets are placed 200 mm away from the wall (position-2) thus signifying an optimum location for the outlets to get highest mixing in a given multi strand tundish.*

Nomenclature

C = Concentration of tracer
C_{av_t} = Average Concentration of the tracer at outlet i, $(i = 1, 2, 3)$
k = Turbulent kinetic energy
p = Pressure
t = Time
t_r = Actual mean residence time of fluid in the vessel, Equation (7)
u = Mean velocity
V = Volume of the tundish

The second author, SKD, gratefully acknowledges the support of the Alexander von Humboldt Foundation for donating him a high end PC on which the present computations were carried out, along with the preparation of the manuscript.

International Journal of Numerical Methods for Heat & Fluid Flow,
Vol. 12 No. 5, 2002, pp. 560-584.

DOI 10.1108/09615530210434296

w'	= fluctuating velocity of w component of mean velocity	τ	= Theoretical mean residence time, Equation (6)
x	= Coordinate for measure of distance	ϕ	= Either k or ε
ρ	= Density of the fluid		
μ	= Co-efficient of viscosity	*Suffix*	
ν	= Kinematic viscosity	i, j, k	= Three Cartesian coordinate directions x, y and z
$\overline{u_i u_j}$	= Average turbulent stress		
ε	= Rate of dissipation of turbulent kinetic energy	d	= Dead volume
		m	= Mixed volume
σ_c	= Turbulent Schmidt number	p	= Plug volume

Introduction

The standard high Reynolds number k-ε turbulence model has been widely used in industrial applications to predict the overall performance of a device. The model has been proved to be very robust and economical from the view point of computer time because of the use of standard wall functions. However, it has been observed that in recirculating flow, the prediction of near wall quantity using the k-ε model does not compare very well with other low Reynolds number models. So for accurate prediction of overall quantity (of mixing, mean residence time, mix volume and dead volume) in a device, modified forms of the standard k-ε model have been developed in the last decade. However, the use of such modified k-ε models has not been made very extensively for industrial cases excepting its validation with simple experiments. It has been the main motivation of the present work to use the standard k-ε model of Launder and Spalding (1972) along with its two modifications, RNG (Yahkot and Orszag 1992) and Lam and Bremhorst (1981) to predict the mixing in a single and multi strand (multi exit) tundish which is a reasonably complex shaped device (when fitted with baffles, advanced pouring box and shroud) and plays a very important role in the steel industries for casting quality steel. The tundish is the last device in the sequential operation of steelmaking where final controls can be made to improve the quality of steel and decide on its final chemistry. Hence, fluid flow and mixing in a tundish have been studied by many authors, both numerically and experimentally (Debroy and Sychterz, 1985; Tacke and Ludwig, 1987; Yeh *et al.*, 1992; Szekely *et al.*, 1987; Xintian *et al.*, 1992; He and Sahai, 1987; Madias *et al.*, 1999). However, all the mathematical models of the past have used the standard k-ε model to solve the velocity field in the tundish and predict tracer concentration henceforth. The effects of various turbulence models on mixing have not been reported or compared with experimental measurements made in a tundish. Moreover, the effect of outlet positions on mixing, for a multi strand tundish have not been reported by using various turbulence models. Jha *et al.* (2001) have reported the effect of outlet positions on mixing by using the standard k-ε model and the present work is an extension of their work where various turbulence models are tried along with different outlet positions to examine their effect on mixing.

Physical description of the problem

The geometry of the multi strand tundish is shown in Figure 1a along with the three outlets. Half of the tundish is shown because of the symmetry about the inlet plane. The depth of the tundish and the bottom pad are 572 mm and 280 mm respectively with the size of the inlet as 25 mm × 50 mm and all other dimensions are shown in a plan view in Figure 2, which completes the detail geometrical description of the industrial size tundish taken for the mathematical simulation. It is to be noted that the outlets (15 mm × 15 mm) are drilled through the bottom pad at the positions shown and they have a length of 280 mm the same as the thickness of the bottom pad. The positions of the outlets are measured from the bottom wall and the exact locations of the outlets are shown in Figure 2. At position-1 the outlets are placed 50 mm away from the wall while at position-3 the outlets are 300 mm away from the wall. Mixing in the tundish is studied by injecting a dye through the inlet stream for a very short time and then computing the mass concentration of the dye in the entire tundish as a function of time. The intention is to compute the ratio of mixed to dead volume and the mean residence time in the tundish by using various turbulence models, which are regarded as the main parameters for deciding the effective utilization of the tundish volume and hence mixing in the tundish. Also the response of the dye at all the outlets is monitored which in turn helps to compute the mixed and dead volume as well as the mean residence time (Jha *et al.*, 2001; Szekely and Themelis, 1971; Levenspiel, 1972). The objective is to find out a suitable location of the outlets, which can induce the highest possible mixing in the tundish and to study the effect of the outlet positions by using different turbulence models. Before proceeding to compute the above, a detail computation on a single strand (exit) tundish is done (Figure 1c) with all the three turbulence models, which have been compared with the experimental measurement of Singh and Koria (1993). Experimental measurements for a multi exit tundish have not been found so far in literature for which reason comparisons could not be made with a multi exit tundish.

Mathematical formulation and assumptions

The flow field in the tundish is computed by solving the mass and momentum conservation equations in a boundary fitted coordinate system along with a set of realistic boundary conditions. The tundish boundary does not conform to a regular Cartesian system, because it is an inclined wall delta shape tundish, so the use of BFC was made to solve all the conservation equations. The species continuity equation is solved in a temporal manner to capture the local variation of the concentration of the dye in the tundish. The free surface of the liquid in the tundish was considered to be flat and the slag depth was

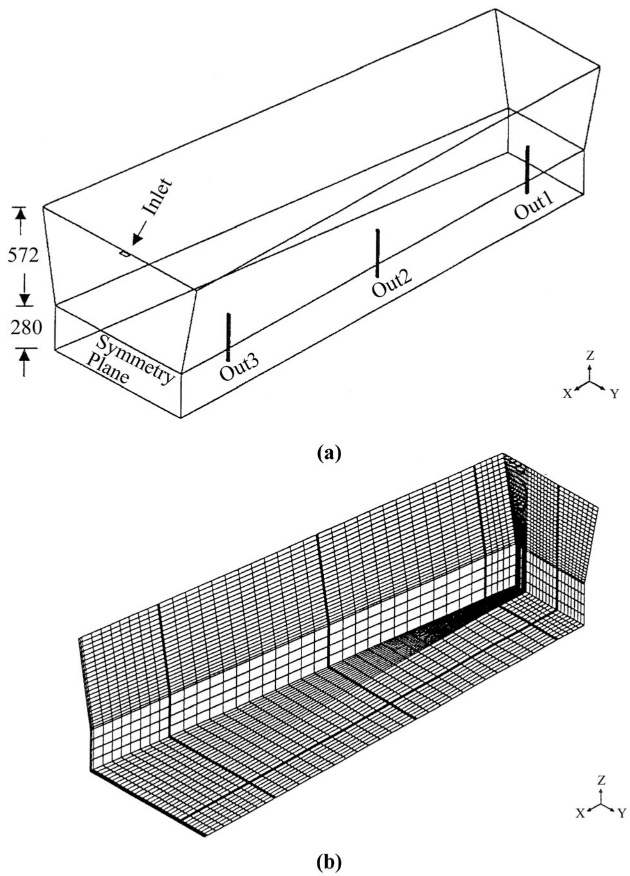


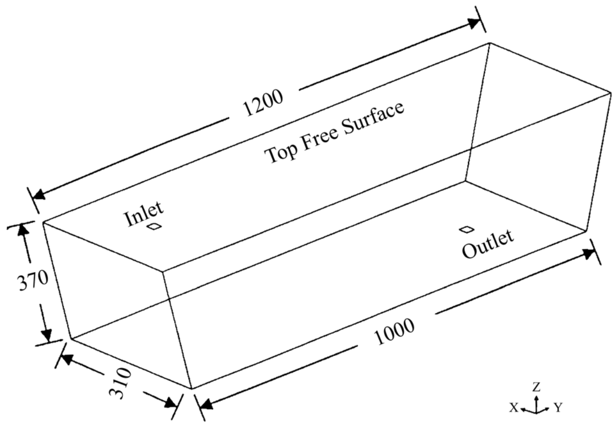
Figure 1.
(a) Geometry of the six strand billet caster tundish about the symmetry plane with the inlets and outlets drilled through the bottom pad. (b) The billet caster tundish with the boundary fitted grid lines shown on the outermost surfaces. (c) Geometry of the tundish used for experiment by Singh & Koria (All dimensions are in mm.) (d) Tundish of Figure 1c with boundary fitted grid lines on the outermost surfaces

(*Continued*)

considered to be insignificant. With these two assumptions the flow field was solved with the help of the following equations (in tensorial form) with three turbulence models (k-ε, k-ε-RNG and Low Re k-ε Lam Bremhorst). The effect of natural convection is ignored in the tundish because the ratio, $\mathrm{Gr}/\mathrm{Re}^2 = 0.044\,\Delta T$ (Lopez-Ramirez *et al.*, 2000), where ΔT, the driving force for natural convection is the temperature difference between the liquid steel at the top free surface of the tundish and the bulk temperature of the liquid, which is much less than unity for all the cases that are computed here.

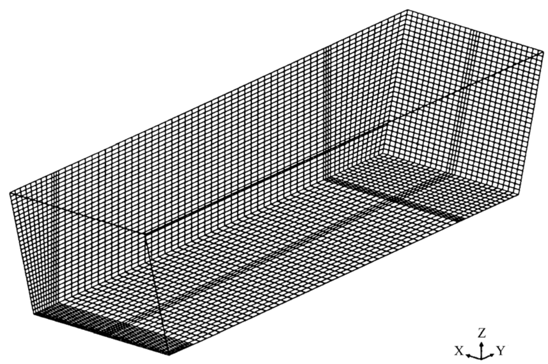
Governing equations
Continuity

$$\frac{\partial}{\partial x_i}(\rho U_i) = 0 \qquad (1)$$



(c)

Note: All dimensions are in mm



(d)

Figure 1.

Momentum

$$\frac{\mathrm{D}(\rho U_i)}{\mathrm{D}t} = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j}\left[\mu\left\{\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right\} - \overline{\rho u_i u_j}\right] \quad (2)$$

Turbulent kinetic energy

$$\frac{\mathrm{D}(\rho k)}{\mathrm{D}t} = D_k + \rho P - \rho\varepsilon \quad (3)$$

Rate of dissipation of k

$$\frac{\mathrm{D}(\rho\varepsilon)}{\mathrm{D}t} = D_\varepsilon + C_1 f_1 \rho P \frac{\varepsilon}{k} - C_2{*} f_2 \frac{\rho\varepsilon^2}{k} \quad (4)$$

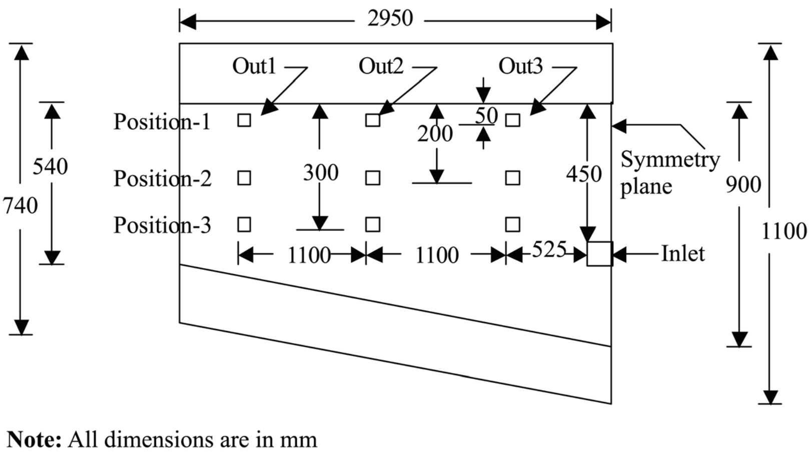


Figure 2. Top view of the tundish about the symmetry plane with the location of the outlets at the bottom plane (All dimensions are in mm)

Concentration

$$\frac{\partial}{\partial t}(\rho C) + \frac{\partial}{\partial x_i}(\rho u_i C) = \frac{\partial}{\partial x_i}\left(\frac{\mu_{\text{eff}}}{\sigma_c}\frac{\partial C}{\partial x_i}\right) \tag{5}$$

Where,

$$\overline{u_i u_j} = \frac{2}{3}k\delta_{ij} - \nu_t\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right)$$

$$\nu_t = C_\mu f_\mu k^2/\varepsilon, \qquad \mu_{\text{eff}} = \rho\nu_t + \mu$$

$$D_\phi = \frac{\partial}{\partial x_j}\left[\left(\mu + \frac{\mu_t}{\sigma_\phi}\right)\frac{\partial \phi}{\partial x_j}\right], \qquad P = -\overline{u_i u_j}\frac{\partial U_i}{\partial x_j}$$

Constants (k-ε model)

$$C_1 = 1.44, \qquad C_2{*} = C_2 = 1.92, \qquad \sigma_{\text{c}} = 1.0$$

$$\sigma_k = 1.0, \qquad \sigma_\varepsilon = 1.3, \qquad f_1 = f_2 = f_\mu = 1, \qquad C_\mu = 0.09$$

RNG model

$$C_2{*} = C_2 + \frac{C_\mu \eta^3(1 - \eta/C_3)}{1 + C_4\eta^3},$$

$\eta = Sk/\varepsilon$, $S = \sqrt{2S_{ij}S_{ij}}$ = modulus of the mean rate-of-strain tensor

$$S_{ij} = \frac{1}{2}\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right)$$

$$C_1 = 1.42, \quad C_2 = 1.68, \quad C_3 = 4.38, \quad C_4 = 0.012, \quad C_\mu = 0.085$$

$$\sigma_c = 1.0, \quad \sigma_k = 1.0, \quad \sigma_\varepsilon = 1.3, \quad f_1 = f_2 = f_\mu = 1$$

Lam-Bremhorst model

$$f_\mu = [1 - e^{(-0.0165 \ \mathrm{Re}_n)}]^2\left(1 + \frac{20.5}{\mathrm{Re}_t}\right), \qquad f_1 = 1. + (0.05/f_\mu)^3,$$

$$f_2 = 1. - \exp(-\mathrm{Re}_t^2), \qquad \mathrm{Re}_n = \frac{\sqrt{k} \times Y_n}{\nu}, \qquad \mathrm{Re}_t = \frac{k^2}{\nu\varepsilon}$$

Y_n = Distance to the nearest wall

$$C_1 = 1.44, \quad C_2{*} = C_2 = 1.92, \quad \sigma_c = 1.0,$$

$$\sigma_k = 1.0, \quad \sigma_\varepsilon = 1.3, \quad C_\mu = 0.09$$

Computation of mixed and dead volume (Jha *et al.*, 2001; Szekely and Themelis, 1971; Levenspiel, 1972)

$$\text{Theoretical residence time } \tau = \text{Volume of tundish/(Volumetric flow rate)} \tag{6}$$

$$\text{Actual residence time } t_\mathrm{r} = \frac{\sum C_{\mathrm{av}_i} t_i}{\sum C_{\mathrm{av}_i}},\ i = 1, 2, 3 \text{ (for the three outlets)} \tag{7}$$

In Equation (7) the integration is carried over a time span of 2τ with an equal interval of time step.

$$\text{Average break through time, } t_\mathrm{p} = \text{First appearance of tracer at the exits (time to be averaged for multi exits)} \tag{8}$$

In case of a multi strand tundish, the value of t_p will vary from one outlet to the other. The tracer will appear suddenly at the outlet, which is placed nearest to the inlet. So the value of t_p will be very small for this outlet where as for other outlets t_p will have a higher value. In order to model the break through time for the entire tundish, the individual values of t_p for all the outlets are added and an average value of t_p is taken to compute the plug volume. The dead volume is computed from Equation (9) after computing the theoretical and actual mean residence time from Equation (6) and (7) respectively. The mixed volume is computed from Equation (11) after the dead and plug volumes are computed.

$$\text{Fraction of dead volume,} \quad V_d/V = 1 - t_r/\tau \tag{9}$$

$$\text{Fraction of plug volume,} \quad V_p/V = t_p/\tau \tag{10}$$

$$\text{Fraction of mixed volume,} \quad V_m/V = 1 - V_p/V - V_d/V \tag{11}$$

Boundary conditions

Boundary conditions can be well visualized with reference to Figure 1a, c The symmetry plane is given a symmetry boundary condition, which implies a zero gradient condition for all variables normal to that plane. The walls were set to a no slip condition and the turbulent quantities were set from a log law wall function for the k-ε and k-ε -RNG models. The following "logarithmic law of the wall" (Ferziger and Peric, 1999) was utilized to compute the value of $k(k_p)$ and $\varepsilon(\varepsilon_p)$ at the first cell in contact with the wall by considering the production and dissipation of turbulent quantities to be in local equilibrium near the wall.

$$\frac{\rho u_p k_p^{1/2} C_\mu^{1/4}}{\tau_w} = \frac{1}{\kappa}\ln(Ez^+), \quad \text{where } z^+ = \frac{z_p k_p^{1/2} C_\mu^{1/4}}{\nu}, \quad \mathrm{E} = 8.6 \text{ and } \kappa = 0.41$$

$$\varepsilon_p = \frac{C_\mu^{3/4} k_p^{3/2}}{\kappa z_p}$$

The same wall function is used in the k-ε -RNG model for the computation of the near wall turbulent quantities. It can be noticed from the above equation that the first node distance from the wall, z_p influences the near wall turbulent quantities. The influence of z_p on mixing parameters (V_m/V, V_p/V and V_d/V) is studied in the present computation in order to arrive at a suitable grid distribution near the wall which can predict more accurate results for mixing.

For the Lam-Bremhorst model $k = 0$ and a normal gradient of ε to the wall was set to 0. At the inlet, the velocity of the incoming jet was set to a prescribed value of 1.4 m/s (1.61 ton/min of liquid steel) with a turbulent intensity of 2 per cent. The intensity of turbulence is defined here to be $I = (\sqrt{\overline{w'^2}}/w_{inlet})$, from which the value of k at the inlet can be prescribed as $k = 0.5(I \times w_{inlet})^2$. The value of ε at the inlet is computed from the relation $\varepsilon_{inlet} = (C_\mu^{3/4} k_{inlet}^{3/2}/0.1H)$, where H is the hydraulic radius of the inlet pipe (Launder and Spalding, 1972).

The top surface of the tundish was taken to be a free surface where a zero shear stress condition was applied according to references (Tacke and Ludwig, 1987; Szekely *et al.*, 1987; Illegbusi and Szekely, 1989). The bottom of the tundish was treated like a wall where no slip conditions were used for the velocity. At the outlets a fixed pressure of 0 Pa (relative to the ambient) was applied. The wall of the tundish was considered to be impervious to the dye, so a zero gradient condition for the dye was used on the walls. At the outlet and at the free surface also zero gradient conditions for the dye were used (Illegbusi and Szekely, 1989; Illegbusi and Szekely, 1988). At the inlet the concentration of the dye was kept from 1 to 5 seconds after which the concentration was kept at zero. 5 seconds is normally very short compared to the mean residence time of the tundish so the influx of the dye during its travel is not likely to change the local velocity field as the mass influx of the dye is also very small (Szekely and Themelis, 1971).

Method of solution

The set of partial differential Equations (1)–(5) was solved with the help of the above boundary conditions numerically in a finite volume technique using the educational version of the CFD software Phoenics. The partial differential equations were integrated over a control volume to find out the fluxes (of mass and momentum as well as that of the dye) through all the faces, and the flux balance is made over all the control volumes, which yield a set of linear algebraic equations. The set of algebraic equations is solved by the tridiagonal matrix (TDM) method for momentum and by a whole field solver, taking one from the family of conjugate gradients for the pressure correction equation. The species continuity equation is solved at each and every time step using the TDM matrix method once the steady state solution for the momentum equations is obtained. The solutions are said to have converged when the whole field normalized residuals for each of the velocity components and mass fall below unity. A false time step relaxation of 0.5 was used for all the variables for faster convergence. Control volumes (CV) of $66 \times 27 \times 30$ $(X \times Y \times Z)$ were used for the computation of the single strand bare tundish for the k-ε and RNG models. By changing the control volumes to $76 \times 33 \times 35$, it was observed that the changes in the mixed and dead volumes were less than 0.2 per cent. For the Lam-Bremhorst model control volumes of $73 \times 35 \times 33$ $(X \times Y \times Z)$ were used which could yield an $z+$ value of nearly 1 or somewhere less than 1 near the tundish wall. Increasing the CVs by another 10 in all directions did not improve the mixed volume and the mean residence time, even by 0.1 per cent. A typical run for a k-ε model takes about 14 hours for the solution of the velocity field and 14 hours for the solution of the concentration field, whereas the Lam-Bremhorst model takes 40 hours and 22 hours for the respective solutions of the velocity and concentration field. From the temporal variation of concentration the actual mean residence time and all other times was found out by simple integration (Equation (7)) after which the ratio of mixed to dead volume could be found out. For the computation, the density of liquid steel was taken to be 7100 kg/m^3 all through the volume and the kinematic viscosity (Mazumdar and Guthrie, 1999) to be $0.913 \times 10^{-6}\,\mathrm{m}^2/\mathrm{s}$.

Results and discussions

The flow field in the six strand (multi exit) tundish was obtained by solving the Navier Stokes equations numerically and then the tracer dispersion was computed by injecting some dye into the inlet. From the tracer dispersion curve the mixed volume and the dead volume were computed as per Equations (9)–(11). The analysis of mixing was done with respect to the ratio of mixed to dead volume for different geometrical positions of the outlet by using three different turbulence models. The mixed and dead volumes are direct indices of mixing in a tundish. If the mixed volume is large that means more of the tundish volume is utilized in mixing the fluid. In a similar way it can be said that if the dead volume is low then most of the volume of the tundish is utilized by the fluid for mixing. So a ratio of mix to dead volume (Singh and Koria, 1995) and the mean residence time are better parameters to describe the mixing in a tundish as a function of other geometrical parameters. In the present study the effect of outlet positions on mixing has been carried out. We will discuss the temporal variation of tracer concentration, then the ratio of mixed to dead volume (V_m/V_d) and the mean residence time as a function of outlet positions.

Validation with experiment

Singh and Koria (1993) have done the experiment for a single inlet- single outlet tundish in which they have measured the tracer concentration with time at the outlet. The geometry of their tundish is shown in Figure 1c and the computational cells used in the present computation are shown in Figure 1d. In this experiment the bath height was kept at 260 mm and accordingly the same height was used for the computation where the free surface boundary condition was applied. Figure 3a shows the temporal variation of the tracer concentration (non-dimensional) with non-dimensional time and its comparison of three turbulence models with the one of the experiment. It can be seen from the figure that the tracer concentration has two peaks in the experiment (one at $t = 0.15$ and the other at $t = 0.42$) and all the three turbulence models are able to predict both peaks but they have their own delays in time while predicting them.

When the tracer is first added at the inlet it moves with the flow field towards the outlet due to the steady velocity field present in the tundish. It takes little time to reach the outlet and that can be seen clearly in Figure 3a when the concentration just starts to rise from a value of zero. The concentration at the outlet then increases with time due to plug flow present in the tundish. A sharp increase in the tracer concentration shows that mixing has not taken place in the tundish because the tracer that has been added has just

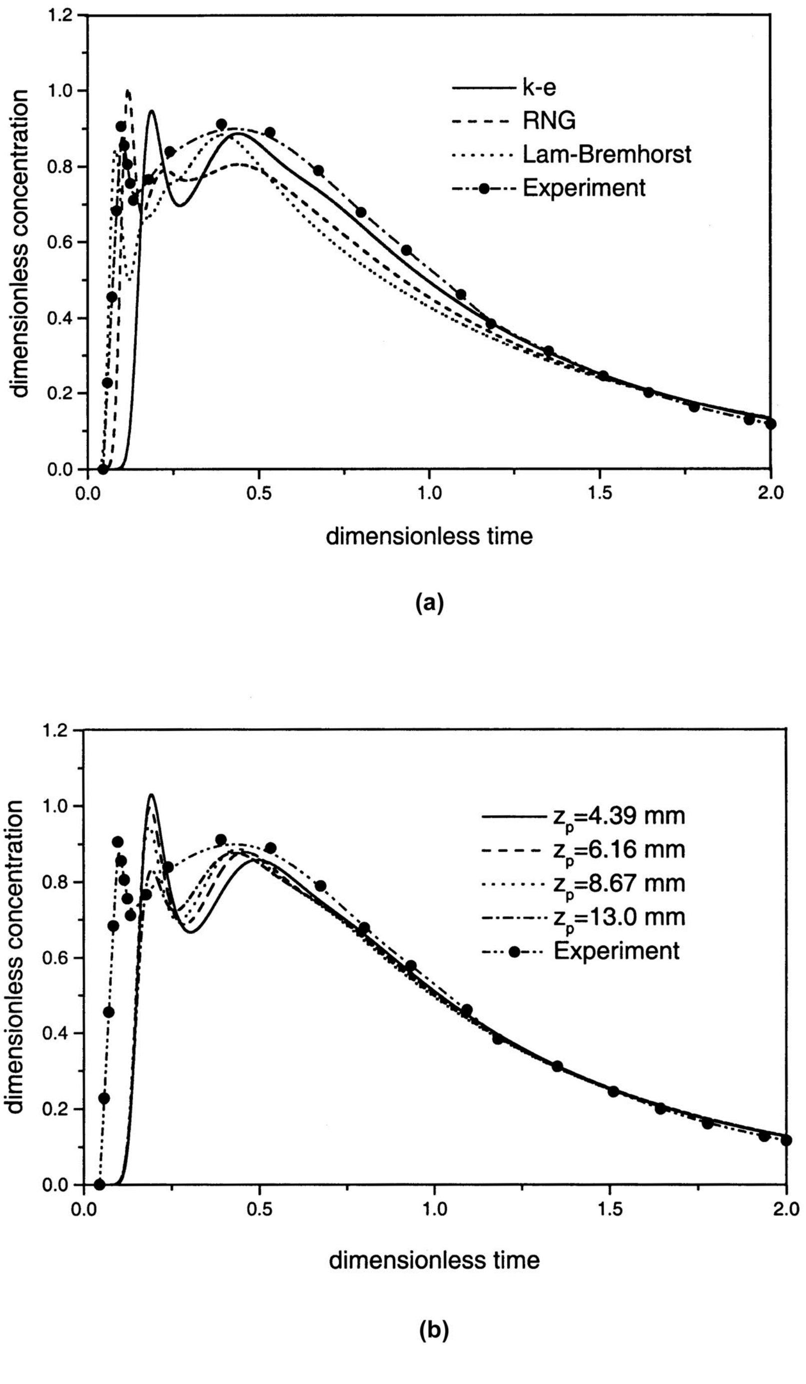


Figure 3.
(a) Temporal variation of tracer concentration at the outlet of a single strand tundish: a comparison between experiment and various turbulence models. (b) Temporal variation of tracer concentration at the outlet of a single strand tundish: a comparison between experiment and the k-ε turbulence model with different values of Z_P

found its way to the outlet for which there is a sudden jump in the concentration at the outlet. If there were mixing then the change in the concentration at the outlet could be gradual, which is seen to be happening at a later time ($t > 0.5$). However after the initial peak, the tracer concentration falls suddenly and then gradually increases to another peak after which it slowly decreases with time. This happens because after the sudden release of tracer material at the outlet there is no tracer present around the outlet for which the concentration suddenly falls. However after a while the fluid brings in some more tracer which has the chance to be mixed by the rebounded fluid from the wall for which the concentration again increases and attains a peak value. This time the tracer concentration does not increase as suddenly as it does the first time. After the second peak the tracer slowly goes out of the system for which the concentration slowly falls with time and after about 3 times the mean residence time, the concentration falls to nearly zero. So it can be found from this experimental observation that mixing has really taken place after a non-dimensional time of 0.2 from when the rise in tracer concentration has become gradual as well as its fall.

It can be seen that the initial rise in the tracer concentration is well predicted by the Lam-Bremhorst model as well as the first peak. Also the k-ε RNG model is capable of predicting the initial rise well, along with the prediction of the first peak. But the standard high Reynolds number k-ε model shows a delay in predicting the first tracer appearance although it predicts the magnitude of the first peak value well compared with the other two models. In the k-ε model, it has been observed that the value of k remains high all through the flow field compared with the other two models. Hence, the turbulent diffusion remains high; as a result, mixing becomes high and the tracer appears late at the outlet. In the RNG model the extra source term in the dissipation equation (4) increases the rate of dissipation near the wall and causes the turbulent kinetic energy to remain low for which the turbulent diffusion remains low causing mixing to be relatively low in comparison with the standard k-ε model. The same effect also comes through the low Reynolds number Lam-Bremhorst model where the overall value of turbulent kinetic energy in the near vicinity of wall remains low, causing turbulent diffusion of the tracer concentration to be low. So the RNG as well as the Lam-Bremhorst model predict the first appearance of tracer to be quicker than the k-ε model alone. However, after $t > 0.5$ the k-ε model predicts a better match in concentration with time compared with the other two models simply because the k-ε model produces more turbulent kinetic energy, which aids diffusional mixing better than the other two models. After a time of 1.5 all the three turbulence models predict the same concentration with time.

It was suspected that the near wall turbulent quantities computed from the logarithmic wall function could be producing higher values of k for the k-ε turbulence model for which the first appearance of tracer at the outlet is delayed compared with the experimental observation. So in order to examine the effect of the "log law wall function" on mixing, the first node distance from the bottom wall (z_p) was changed according to the suggestion given by Chakraborty and Sahai (1991). Figure 3b shows the effect of near wall node on the temporal variation of tracer concentration at the exit of a single strand tundish (Singh and Koria). The near wall node distance, z_p was varied from 4.39 mm (maximum $z^+ = 18$) to 13 mm (maximum $z^+ = 40$) in the k-ε turbulence model but the mean residence time was changed only from 444.51 sec to 445.7 sec respectively. It can be seen from Figure 3b that the first peak in the concentration curve is predicted little higher when z_p is 4.39 mm and subsequently with the increase of z_p to 13 mm the peak decreases. Both the peaks are predicted very near to the experimental observation when z_p has a value of 8.67 mm (maximum $z^+ = 30$). But the initial delay in appearance of the tracer is nearly the same for all the cases of z_p studied here. From Table I, a comparison of the various mixing parameters can be read for different turbulent models. It can be concluded that the grid distribution, with a near wall node located at 8.67 mm is more suitable for predicting the experimental observation.

Table I shows a comparison of the bulk properties for the tundish with all the three turbulence models. It can be seen from the table that the mean residence time is well predicted by k-ε model (all the cases of z_p predicts well) which favorably compares with the experimental value as well as the ratio of mixed to dead volume of the tundish, V_m/V_d. The mixed volume (per cent volume of the entire tundish) is under-predicted by k-ε model where as RNG and Lam-Bremhorst model predict the mixed volume closer to experiment while in case of dead volume, k-ε model predicts closer to experimental value as compared with the overpredicted value by the RNG and the Lam-Bremhorst model. It can be concluded from Table I that the k-ε model is well capable of predicting the mean residence time and V_m/V_d closer to experimental observation in a bare tundish although the flow field inside a tundish is highly recirculating. Although the k-ε model does not predict the variation of temporal concentration well at the beginning, the initial mismatch in concentration with the experimental observation does not influence the prediction in mean residence time because the mean residence time is found out by taking the area moment about the concentration axis. So the initial mismatch does not count much in the overall integration, whereas the matching with experimental observation after the initial transience counts much towards predicting the mean residence time. Although the Lam-Bremhorst model predicts the concentration well at the beginning, it does not do that well towards the later part, so the prediction in mean residence time suffers a little when compared with experimental observation.

Effect of outlet positions

The multi strand tundish has 6 outlets at the bottom pad and one inlet. Due to symmetry in the system only the half of the tundish, which has three outlets, is

Properties used for comparison	Experiment of Singh and Koria (1993)	k-ε				k-ε RNG	k-ε Lam-Bremhorst
		$Z_p = 4.39$ mm	$Z_p = 6.16$ mm	$Z_p = 8.67$ mm	$Z_p = 13.0$ mm		
V_m	74.29	69.33	69.11	69.25	69.91	70.33	71.4
V_d	20.14	20.05	20.46	20.5	19.84	23.38	24.82
V_p	5.57	10.61	10.43	10.25	10.25	6.29	3.78
t_r (Sec)	444	444.51	442.2	442	445.7	426	418
V_m/V_d	3.689	3.458	3.378	3.378	3.523	3.008	2.876

Table I.
A comparison of the bulk flow properties obtained from various turbulence models with the experiment

analyzed and their positions can be seen from Figure 2 as well as in all the figures from 4 to 6. When the outlets are at position-1 they are close to the wall and when they are at position-3, they are away from the wall. The tracer concentration through all the outlets (out1, out2 and out3) is plotted in Figure 4(a–c) for the three turbulence models when the outlets are at position-1. Figures 5(a–c) and 6(a–c) describe the tracer concentration through all the outlets when they are at position-2 and position-3 respectively for the three turbulence models.

Figure 4a shows the temporal variation of concentration through all the three outlets when they are at position-1, (near the wall) by using the k-ε model. Similarly Figure 4b,c show the variation of concentration through all the outlets at position-1 by utilizing the RNG and the Lam-Bremhorst model respectively. It can be observed that when the outlets are at position-1, the concentration through out3, nearest to the inlet attains high peak value very quickly after the tracer is injected. Concentration peak through out2 and out3 does not rise much as they are away from the inlet point of tracer injection. A sudden rise in tracer concentration signifies that there is no mixing. The Lam-Bremhorst model predicts a slightly higher peak in out3 compared with the k-ε and RNG model. When the tracer appears at the outlet out2 and out1, by that time they have got a chance to be mixed with surrounding fluid and moreover these outlets do not receive the fluid elements straight from the inlet, so the peak in concentration does not rise to a high value.

When the outlets are shifted to position-2, the peak value in concentration for out3 falls significantly. For outlets, out1 and out2 also the peak values fall in the Lam-Bremhorst model (Figure 5c). But the RNG model shows a higher peak for out2 compared with postion-1. But when the outlets are further shifted to position-3 (Figure 6a–c) the peak values of concentration in out2 increases while for out3 it decreases, excepting the prediction by the RNG model (Figure 6b) compared with position-2. It can be seen from Figures 4–6 that all the three models of turbulence predict similarly the tracer concentration through all the outlets. It can be observed that outlets placed at position-1 definitely do not help better mixing while outlets at position-2 or position-3 do make mixing better. In order to arrive at a better conclusion the bulk flow properties like the mean residence time and the ratio of mixed to dead volume V_m/V_d, are analyzed for the tundish by varying the outlet positions.

Variation of V_m/V_d *and mean residence time*

Figure 7a shows the variation of V_m/V_d when the distance of the outlets from the wall is changing. When the outlets are too close to the wall, V_m/V_d has a small value and as the distance of the outlets increase from the wall, V_m/V_d increases, attains a peak value and then decreases. When the outlets are 200 mm away from the wall V_m/V_d attains the highest value signifying best possible mixing in the tundish. When the outlets move still further away from

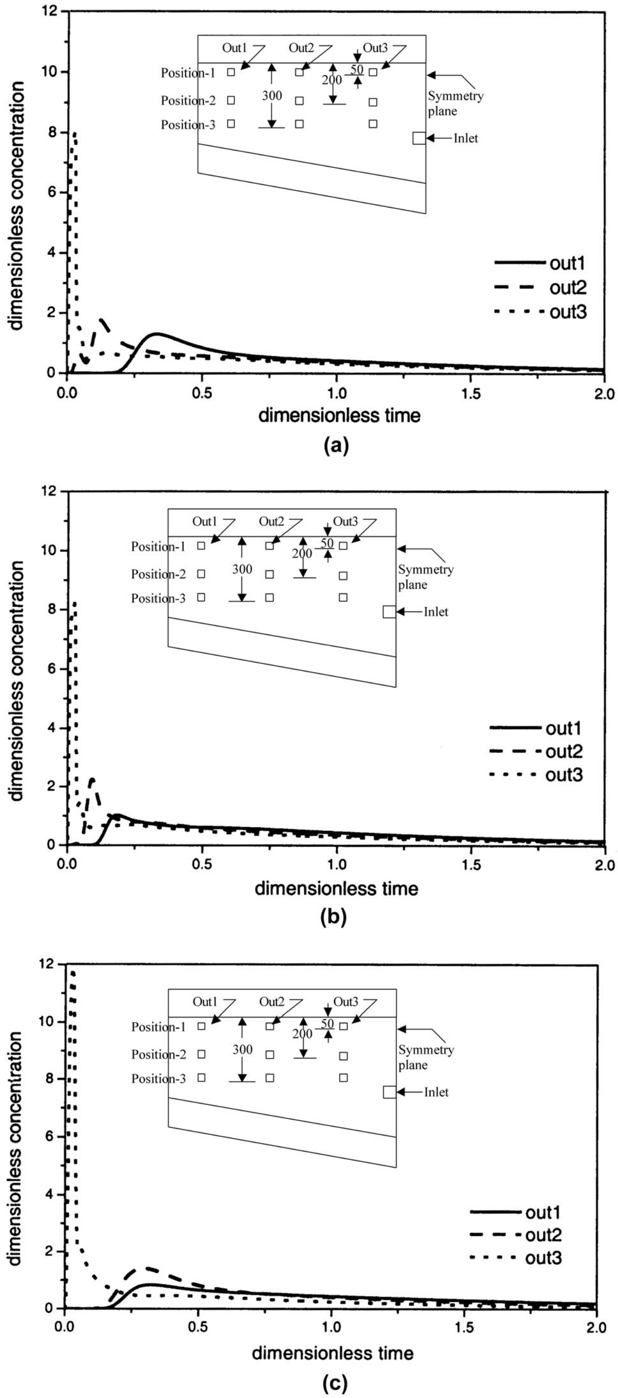


Figure 4.
(a) Variation of concentration with time for the outlets at position-1 (k-ε model). (b) Variation of concentration with time for the outlets at position-1 (RNG model). (c) Variation of concentration with time for the outlets at position-1 (Lam-Bremhorst model)

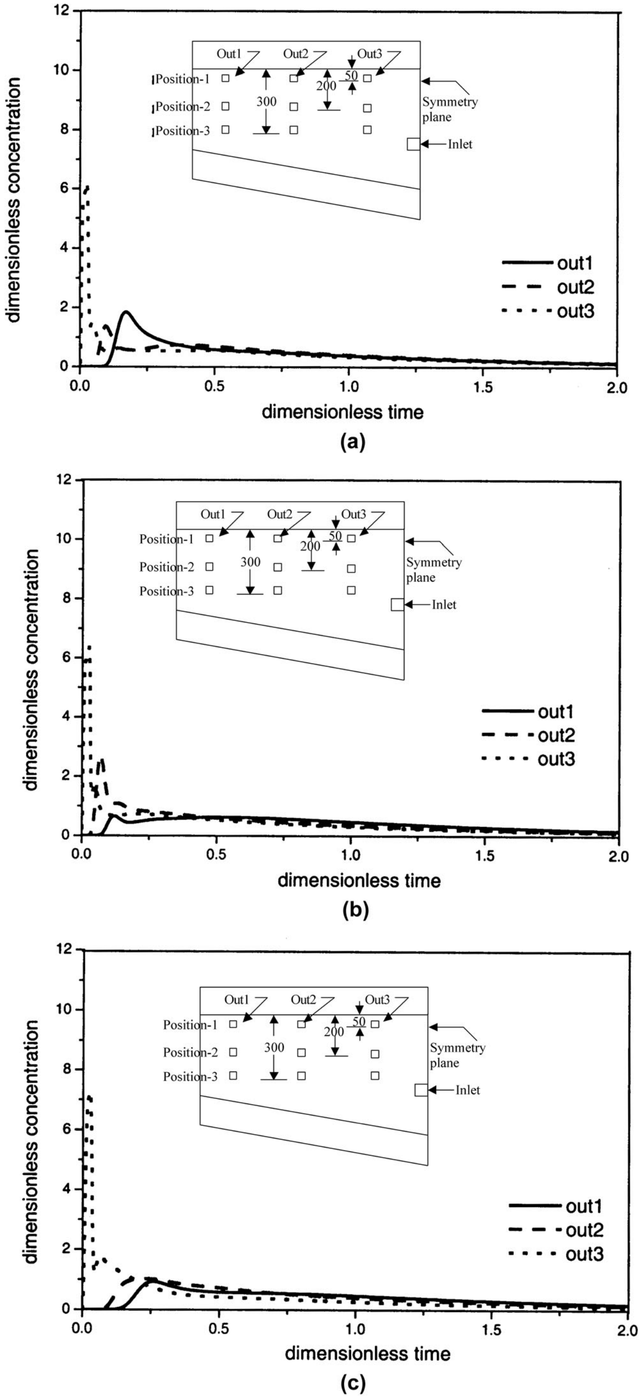


Figure 5.
(a) Variation of concentration with time for the outlets at position-2 (k-ε model). (b) Variation of concentration with time for the outlets at position-2 (RNG model). (c) Variation of concentration with time for the outlets at position-2 (Lam-Bremhorst model)

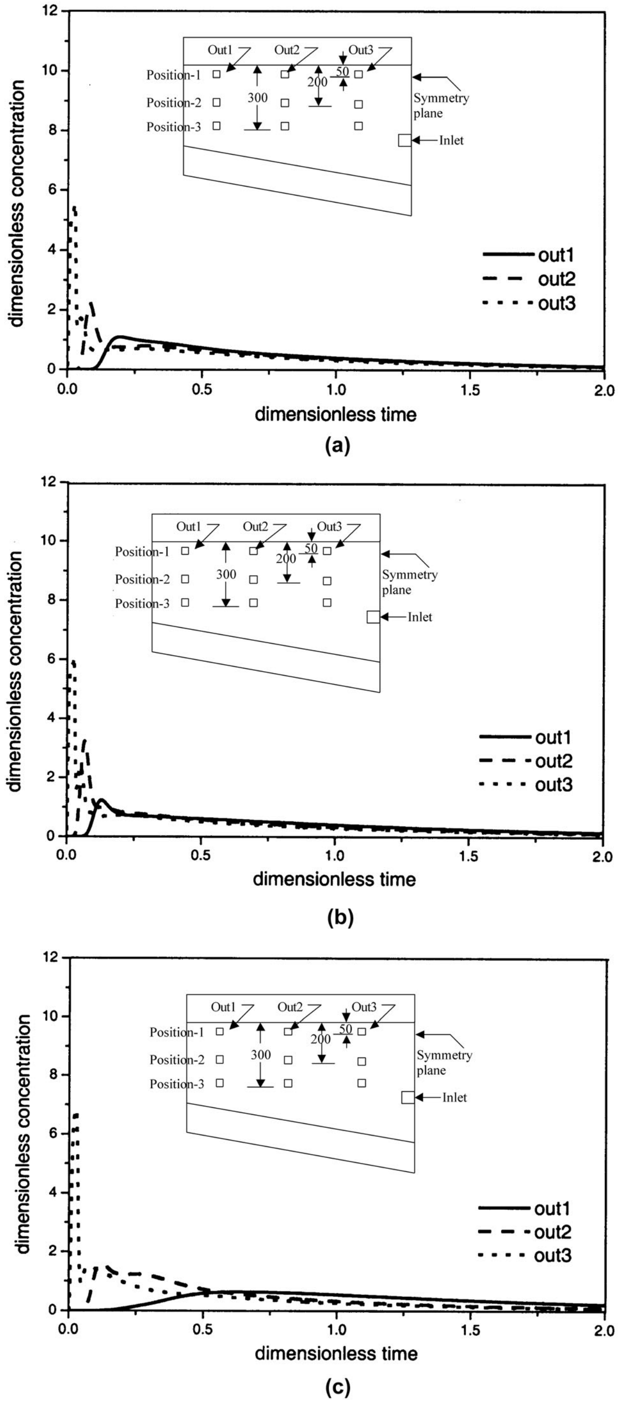


Figure 6.
(a) Variation of concentration with time for the outlets at position-3 (k-ε model). (b) Variation of concentration with time for the outlets at position-3 (RNG model). (c) Variation of concentration with time for the outlets at position-3 (Lam-Bremhorst model)

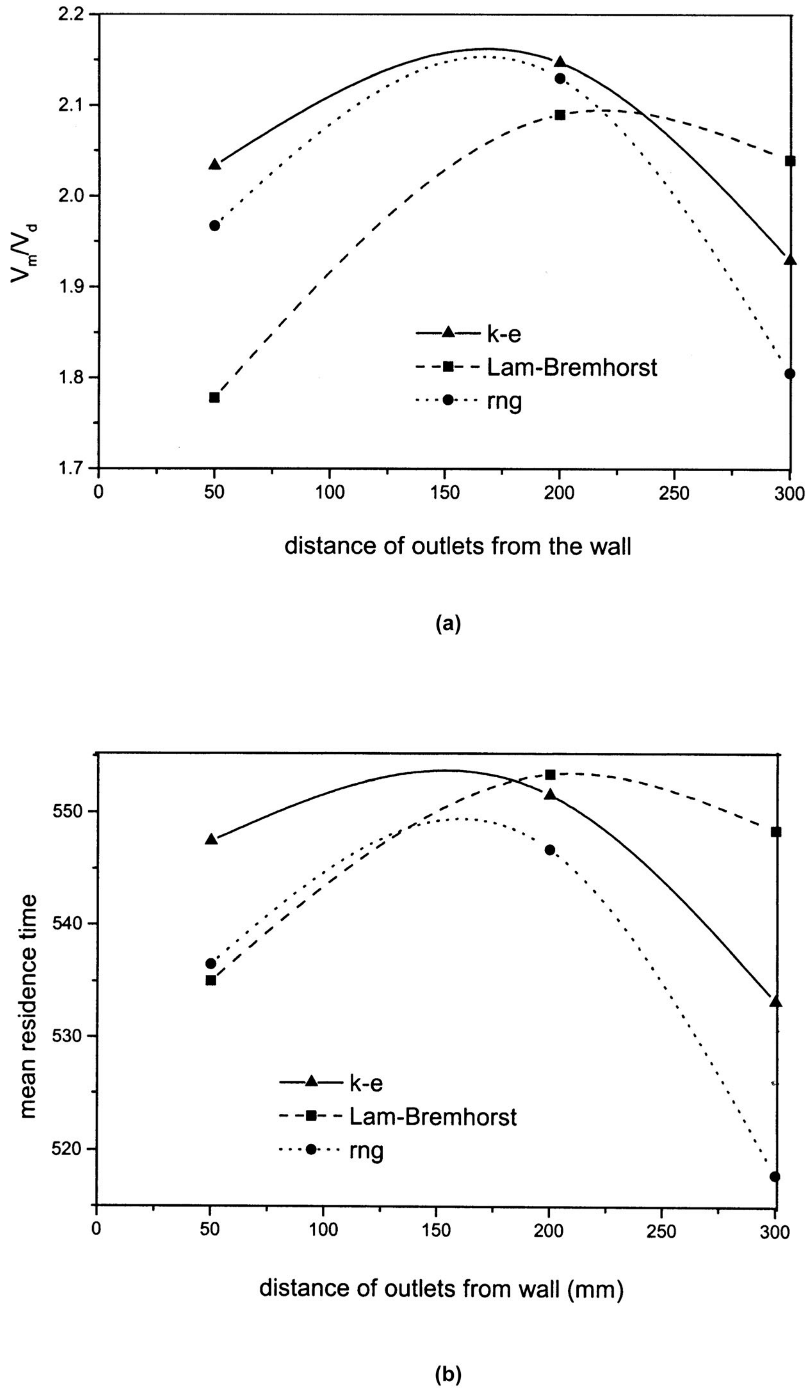


Figure 7.
(a) Variation of the ratio of mix to dead volume as a function of the distance of outlets from the wall for different turbulence models. (b) Variation of mean residence time as a function of the distance of outlets from the wall for different turbulence models

the wall, V_m/V_d decreases because the tracer before dispersing any further finds its way directly to the outlet for the discharge. Thus it can be said that there exists an optimum location for the outlets where best possible mixing can be achieved. All the three turbulence models predict outlet position-2 as the optimum location. The variation of mean residence time with outlet positions in the tundish is plotted in Figure 7b. The variation of mean residence time is also a direct indication of mixing in a tundish. From the figure it is seen that outlet position-2 has the highest mean residence time, hence mixing in the tundish can be better if the outlets are placed at this position.

It can be seen that the RNG and the k-ε model almost predict V_m/V_d very close to each other (with a maximum of 6 per cent deviation from k-ε model) while the Lam-Bremhorst model predicts with a maximum difference of 12 per cent from the k-ε model. However all three models show the same trend in predicting V_m/V_d and the mean residence time (Figure 7b). In the prediction of mean residence time all the three turbulence models are close to each other with, a maximum relative difference of 5 per cent existing between the RNG and the Lam-Bremhorst model.

Analysis of velocity fields

From the analysis of V_m/V_d and mean residence time in Figure 7 it can be said that outlets at position-2 induce better mixing in the tundish. It can be further demonstrated by taking a vertical cross sectional plane through the outlets and by plotting the velocity vector on such a plane. Such velocity vectors can be seen in Figures 8–10. Figure 8a–c show the velocity vector plotted on vertical cross sectional planes, when the outlets are passing through position-1, 2 and 3 respectively, by using the k-ε model while similar plots can be seen in Figure 9 and 10 where the RNG and Lam-Bremhorst models are used. It can be seen from Figure 8b that the recirculation zone is much larger when the outlets are at position-2. Recirculation is weaker at positions-1 and 3 compared with outlets placed at position-2. Similar pictures are obtained from the use of the RNG model. The RNG model also shows that recirculation is much stronger in the tundish when the outlets are at position-2. From the use of the Lam-Bremhorst model it can be seen that recirculation is weaker at all the positions of the outlets. This is the main reason why this model predicts a higher peak in tracer concentration when the outlets are at position-1. Due to lack of recirculation, mixing suffers in the tundish and the tracer does not mix in the bulk of the fluid just after its injection and that is why it suddenly appears at the outlet, causing the concentration to rise very high. It is believed that the present Lam-Bremhorst model dampens the turbulent kinetic energy too much which causes the flow to laminarize, and suppresses turbulent mixing in this particular arrangement of tundish geometry. When the outlets are at position-2 there is a formation of recirculation zone near the symmetry plane and the first outlet, out3. The recirculation persists when the outlets are moved to position-3

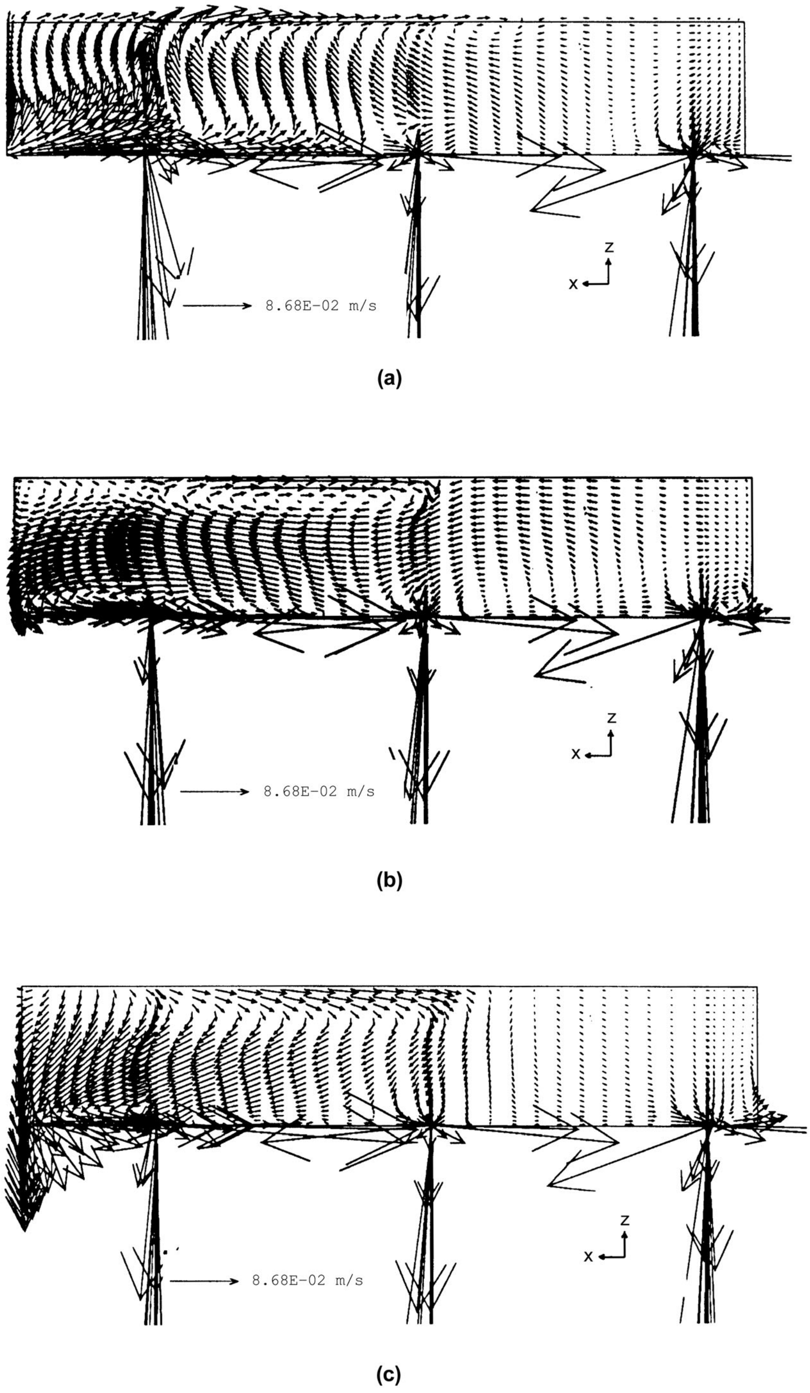


Figure 8.
(a) Velocity field through a vertical cross sectional plane passing through all the outlets at position-1 (K-ε model). (b) Velocity field through a vertical cross sectional plane passing through all the outlets at position-2 (K-ε model). (c) Velocity field through a vertical cross sectional plane passing through all the outlets at position-3 (K-ε model)

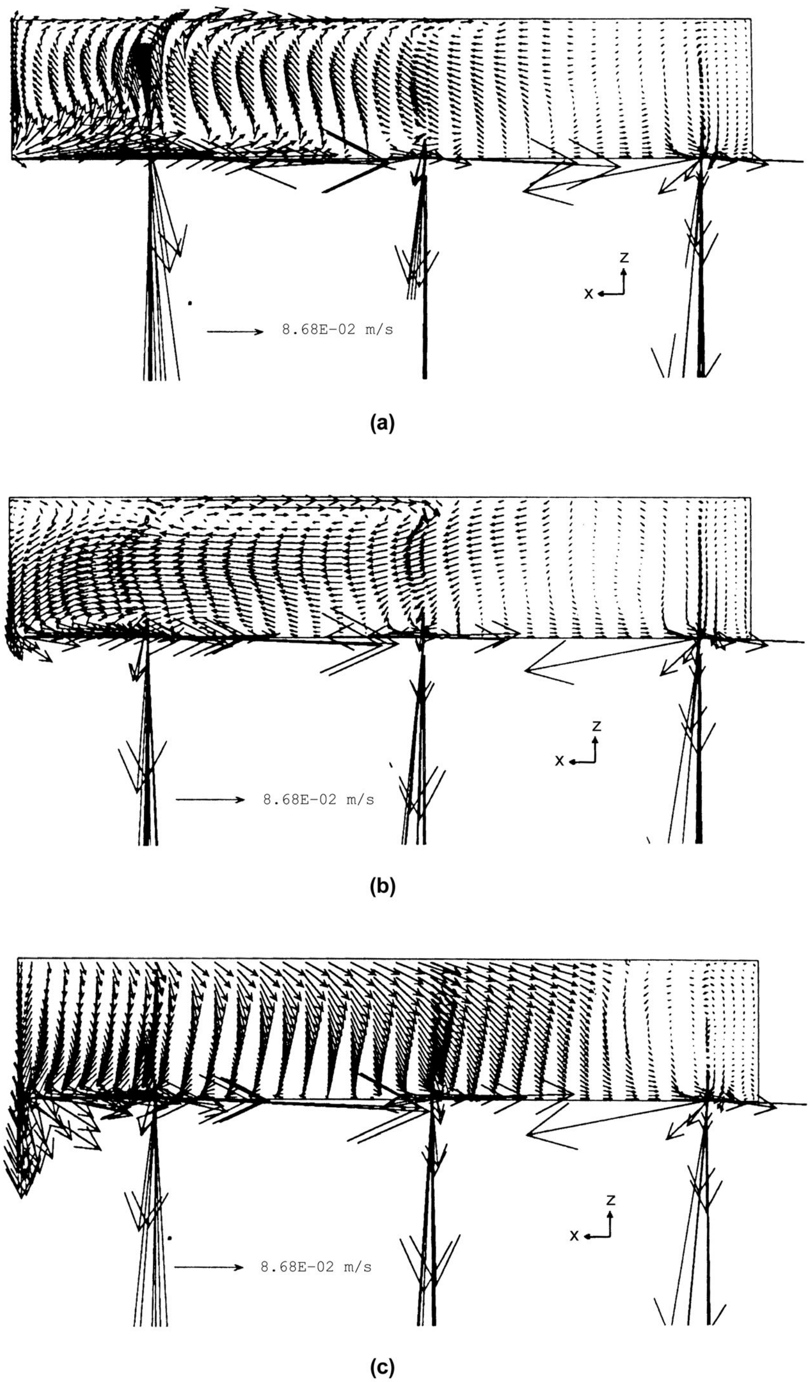


Figure 9.
(a) Velocity field through a vertical cross sectional plane passing through all the outlets at position-1 (RNG model). (b) Velocity field through a vertical cross sectional plane passing through all the outlets at position-2 (RNG model). (c) Velocity field through a vertical cross sectional plane passing through all the outlets at positio-3 (RNG model)

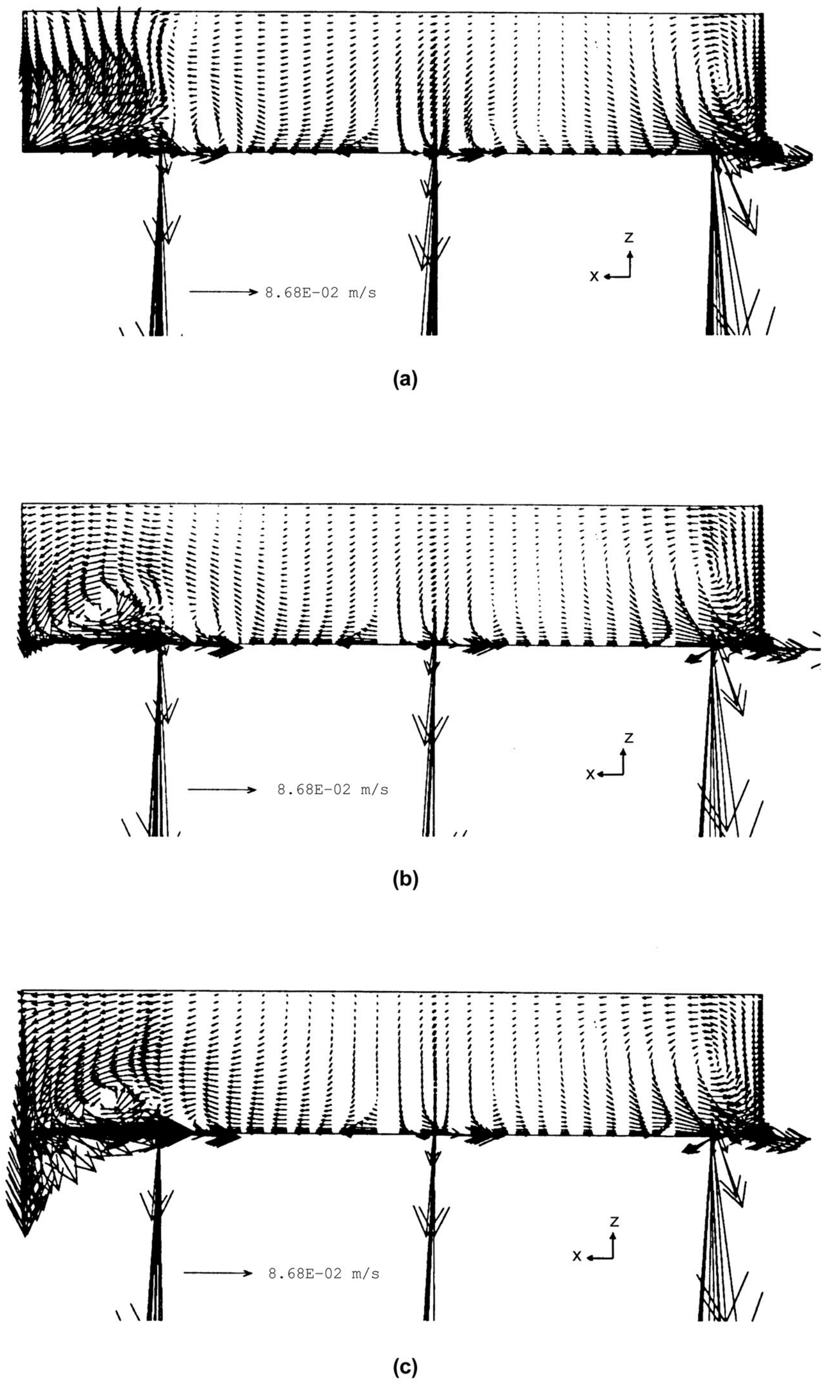


Figure 10.
(a) Velocity field through a vertical cross sectional plane passing through all the outlets at position-1 (Lam-Bremhorst model). (b) Velocity field through a vertical cross sectional plane passing through all the outlets at position-2 (Lam-Bremhorst model). (c) Velocity field through a vertical cross sectional plane passing through all the outlets at position-3 (Lam-Bremhorst model)

and becomes even stronger as it appears from Figure 10c. It should be observed from the k-ε (Figure 8) and RNG (Figure 9) model that they produce almost similar flow fields when the outlets are at position-1 and 2. But when the outlets are at position-3 the velocity field varies a lot between the two models. The structure of the flow field is almost the same for outlet positions-1 and 2 when the Lam-Bremhorst model is used, but the magnitude of velocity is little less compared to the k-ε and RNG model. When the outlets are at position-3, the structure of the flow field predicted by all three models becomes greatly different. When the outlets are at position-3 it is much closer to the inlet. So the velocity field is much stronger there. The inlet jet hits the bottom pad and spreads along the bottom pad towards the outlets, causing a strong shear flow. The strong shear is dampened more by the Lam-Bremhorst and the RNG model compared with the k-ε model. As a result the average turbulent kinetic energy remains higher for the k-ε model compared with the other two turbulence models. A high value of k causes more momentum mixing due to higher diffusion whereas the Lam-Bremhorst model tries to laminarize the flow near the bottom pad. The slow growth of velocity profile near the bottom pad can be well visualized in Figure 10a–c between the outlets out1 and out2, whereas uniform flow field can be seen for the k-ε model in Figure 8c in the same location. Such dissimilarities in the flow field predicted by all the three turbulence models tend to deviate the overall prediction of mean residence time and V_m/V_d from each other.

Conclusions

The mass, momentum and the species conservation equations are solved numerically in a boundary fitted coordinate system comprising a typical industrial size tundish having a through put of 1.61 ton/min. The ratio of the mix to dead volume and the mean residence time are analyzed from the solution of the species conservation equation.

It is found that there exists an optimum location of the outlets where the ratio of mixed to dead volume and the mean residence time is the highest. All the three turbulence models (k-ε, k-ε RNG and k-ε Lam-Bremhorst) predict outlet position-2 as the optimum location.

All the three turbulence models predict the gross flow properties like mean residence time and ratio of mixed to dead volume fairly closely to the accuracy of the experimental measurements while the standard k-ε model predicts the closest value. All the turbulence models (in this study) are able to predict the two peaks in the temporal variation of tracer concentration for a single exit tundish, while the Low Reynolds number Lam-Bremhorst model predicts the initial transience much better than the other two models. It is suggested that k-ε model can be used for tundish flow to predict overall flow properties because it takes less than half the time compared with the Lam-Bremhorst model.

References

Chakraborty, S. and Sahai, Y. (1991), "Role of near wall node location on the prediction of melt flow and residence time distribution in tundishes by mathematical modeling", *Metallurgical Transactions*, Vol. 22B, pp. 429-37.

Debroy, T. and Sychterz, J.A. (1985), "Numerical calculation of fluid flow in a continuous casting tundish", *Metallurgical Transactions*, Vol. 16B, pp. 497-504.

Ferziger, Joel H. and Peric, M. (1999), *Computational Methods for Fluid Dynamics*, Springer pp. 279-86.

He, Y. and Sahai, Y. (1987), "The effect of tundish wall inclination on the fluid flow and mixing", *Metallurgical Transactions*, Vol. 18B, pp. 81-91.

Illegbusi, O.J. and Szekely, J. (1988), "Fluid flow and tracer dispersion in shallow tundishes", *Steel Research*, Vol. 59, pp. 399-405.

Illegbusi, O.J. and Szekely, J. (1989), "Effect of externally imposed magnetic field on tundish performance", *Ironmaking and Steelmaking,* Vol. 16, pp. 110-5.

Jha, Pradeep K., Dash, Sukanta K and Kumar, Sanjay (2001), "Fluid flow and mixing in a six-strand billet caster tundish: a parametric study", *ISIJ Int.*, Vol. 41, pp. 1437-46.

Levenspiel, O. (1972), *Chemical Reaction Engineering*, John Wiley & Sons Inc., New York pp. 253-64.

Lam, C.K.G. and Bremhorst, K. (1981), "A modified form of the k-ε model for predicting wall turbulence", *Trans ASME Journal of Fluids Engineering*, Vol. 103, pp. 456-60.

Launder, B.E. and Spalding, D.B. (1972), *Mathematical Models of Turbulence*, Academic Press, London.

Lopez-Ramirez, S., Morales, R.D. and Romero Serrano, J.A. (2000), "Numerical simulation of the effects of buoyancy forces and flow control devices on fluid flow and heat transfer phenomena of liquid steel in a tundish", *Numerical Heat Transfer*, Vol. 37A, pp. 69-85.

Mazumdar, D. and Guthrie, R.I.L. (1999), "The physical and mathematical modeling of continuous casting tundish systems", *ISIJ Int.*, Vol. 39, pp. 524-47.

Madias, J., Martin, D., Ferreyra, M., Villoria, R. and Garamendy, A. (1999), "Design and plant experience using an advanced pouring box to receive and distribute the steel in a six strand tundish", *ISIJ Int.*, Vol. 39, pp. 787-94.

Singh, S. and Koria, S.C. (1993), "Model study of the dynamics of flow of steel melt in tundish", *ISIJ Int.*, Vol. 33, pp. 1228-37.

Singh, S. and Koria, S.C. (1995), "Study of fluid flow in tundishes due to different types of inlet streams", *Steel Research*, Vol. 66, pp. 294-300.

Szekely, J. and Themelis, N.J. (1971), *Rate Phenomena in Process Metallurgy*, John Wiley & Sons Inc., New York pp. 515-56.

Szekely, J., Illegbusi, O.J. and El-Kaddah, N. (1987), "The mathematical modeling of complex fluid flow phenomena in tundishes", *PhysicoChemical Hydrodynamics*, Vol. 9, pp. 453-72.

Tacke, K.H. and Ludwig, J.C. (1987), "Steel flow and inclusion separation in continuous casting tundishes", *Steel Research*, Vol. 58, pp. 262-70.

Xintian, L., Yaohe, Z., Baolu, S. and Weiming, J. (1992), "Flow behavior and filtration of steel melt in continuous casting tundish", *Ironmaking and Steelmaking*, Vol. 19, pp. 221-5.

Yahkot, Y. and Orszag, S.A. (1992), "Development of turbulence models for shear flows by a double expansion technique", *Phys. Fluids A*, Vol. 4, pp. 1510-20.

Yeh, J.-L., Hwang, W.-S. and Chou, C.-L. (1992), "Physical modeling validation of computational fluid dynamics code for tundish design", *Ironmaking and Steelmaking*, Vol. 19, pp. 501-4.